BC Summer Assignment

Welcome to AP Calculus BC! I look forward to working with you next year. This class will give you a strong foundation for college mathematics and give you a leg up on the competition in many cases. Because of time constraints and the large scope of the BC curriculum, the pace of this course is faster than most high school courses and more similar to a college class. The attached notes pages cover the AP Calculus BC topics that were covered in Honors PreCalculus with a few exceptions. For example, the derivatives of e^x and lnx were not covered in Honors PreCalc. You may notice some other topics are addressed in a different way with different emphasis then they were in PreCalc H.

These example problems/notes are from https://calculus.flippedmath.com/
There are short videos and answer keys on the website for each topic. There are additional practice problems, if needed. We will spend the first week reviewing and I will answer any questions that you may have on this material. At the beginning of the 2nd week of school, we will have a quiz on this material and the completed note/examples in this packet will be turned in to Google Classroom for a grade. Once again, I look forward to working with you next year. Enjoy your summer and please email me if you have any problems or questions.

Sincerely,

Calculus

1.1 Can change occur at an instant?

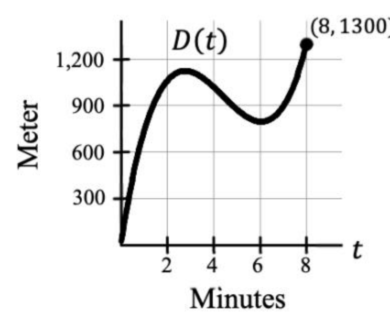
Notes

Write your questions and thoughts here!

The Father(s) of Calculus

Can Change occur at an instant?

1. Mr. Brust's distance from his house is modeled by the function $D(t)$. While riding his bike to the store, he realizes he dropped his wallet and turns around to find it. After finding his wallet, he finishes his ride to the store.



   a. What is his average speed (rate of change) for his trip to the store if he arrives after 8 minutes?

   b. What was his average rate of change between 2 and 6 minutes?

   c. What was his average rate of change between 2 and 3 minutes?

Is it possible to know how fast he was going at an instant?

d. Give an example of how to calculate a rate of change that would give a close estimate to the instantaneous rate of change at $t = 2$.

e. Give a rough estimate of the instantaneous rate of change at $t = 2$.

2. $b(t)$ represents the buffalo population in the United States where t is measured in years since 1800.

   a. What does $b(90)$ represent?

   b. What does $\frac{b(50)-b(0)}{50-0}$ represent?

   c. What does $\frac{b(32)-b(31.999)}{32-31.999}$ represent?

Calculus

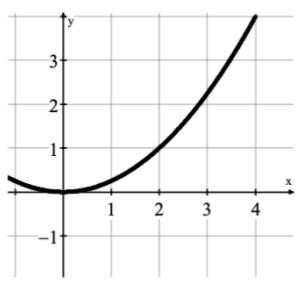
1.2 Defining Limits

Notes

Write your questions and thoughts here!

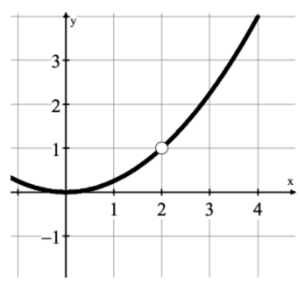
Limits

As x approaches ___, $f(x)$ approaches ___.



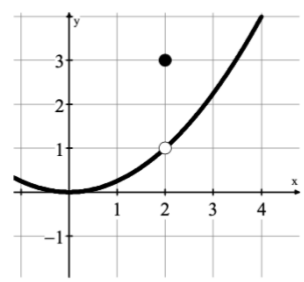
$\lim_{x\to 2} f(x) =$

$f(2) =$



$\lim_{x\to 2} f(x) =$

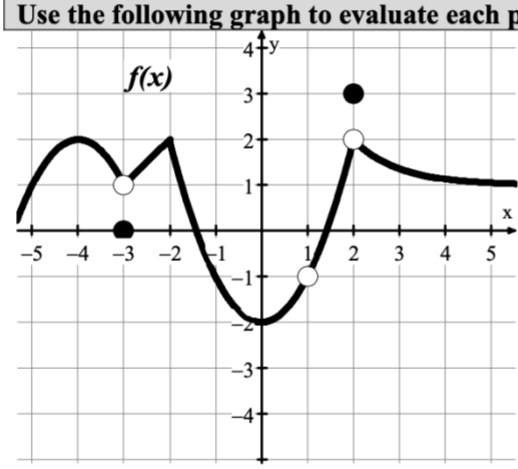
$f(2) =$



$\lim_{x\to 2} f(x) =$

$f(2) =$

Use the following graph to evaluate each problem.



1. $\lim_{x\to 1} f(x) =$
2. $f(-3) =$
3. $\lim_{x\to 2} f(x) =$
4. $f(2) =$
5. $f(1) =$
6. $f(-2) =$
7. $\lim_{x\to 0} f(x) =$
8. $\lim_{x\to -3} f(x) =$

Calculus

1.3 Finding Limits from Graphs

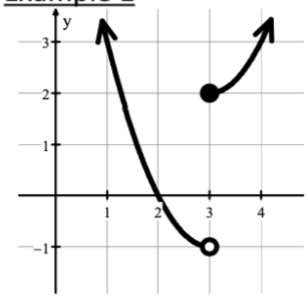
Notes

Write your questions and thoughts here!

<u>What is a *one-sided* limit?</u>

A ***one-sided limit*** is the _______ a function approaches as you approach a given __________ from either the _______ or _______ side.

<u>Example 1</u>



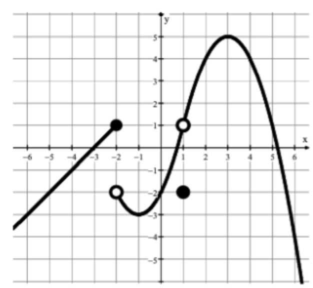
The limit of f as x approaches 3 from the left side is -1. $\quad \lim\limits_{x\to} f(x) =$

The limit of f as x approaches 3 from the right side is 2. $\quad \lim\limits_{x\to} f(x) =$

If the two sides are different? $\quad \lim\limits_{x\to} f(x) =$

<u>Example 2</u>

a. $\lim\limits_{x\to-2^-} f(x) =$	b. $\lim\limits_{x\to-2^+} f(x) =$	c. $\lim\limits_{x\to-2} f(x) =$
d. $\lim\limits_{x\to1} f(x) =$	e. $\lim\limits_{x\to0} f(x) =$	f. $\lim\limits_{x\to3^-} f(x) =$
g. $\lim\limits_{x\to-1} f(x) =$	h. $f(1) =$	i. $f(-2) =$



<u>Example 3</u>

Sketch a graph of a function g that satisfies all of the following conditions.

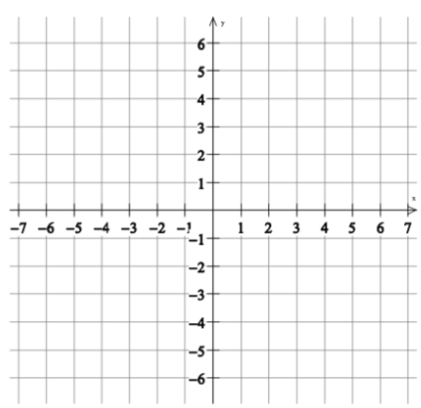
a. $g(3) = -1$

b. $\lim\limits_{x\to3} g(x) = 4$

c. $\lim\limits_{x\to-2^+} g(x) = 1$

d. g is increasing on $-2 < x < 3$

e. $\lim\limits_{x\to-2^-} g(x) > \lim\limits_{x\to-2^+} g(x)$



Calculus

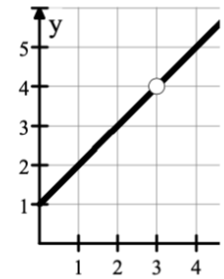
1.4 Finding Limits from Tables

Notes

Write your questions and thoughts here!

Calculator required for part of this lesson!

If we have the graph, it is easy to see the value of $\lim_{x\to 3} f(x) =$



Without the graph, we could use a table of values.

x	2.9	2.99	3.01	3.1
$f(x)$	3.9	3.99	4.01	4.1

1.4 Finding Limits from Tables

Test Prep

13. The table below shows values of the function f at selected values of x. Which of the following is true based on the data from the table?

x	8.9	8.99	8.999	9.001	9.01	9.1
$f(x)$	0.7	0.8	0.999	2.001	2.01	2.3

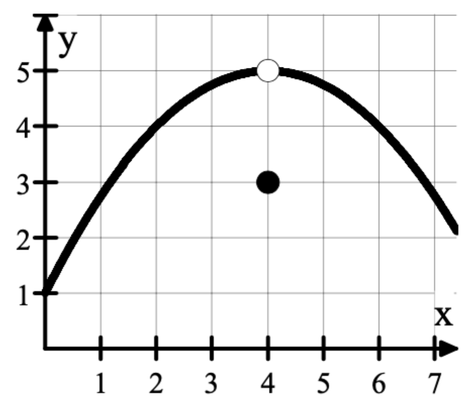
(A) $\lim_{x\to 9} f(x) = 1$

(B) $\lim_{x\to 9} f(x) = 2$

(C) $\lim_{x\to 9^-} f(x) = 2$ and $\lim_{x\to 9^+} f(x) = 1$

(D) $\lim_{x\to 9^-} f(x) = 1$ and $\lim_{x\to 9^+} f(x) = 2$

14. The graph of the function f is shown to the right. The value of $\lim_{x\to 4} 2\cos(f(x))$ is



(A) 0.567

(B) -1.307

(C) -1.979

(D) Does not exist

15. If $[x]$ represents the greatest integer that is less than or equal to x, then $\lim_{x\to 0^-} \frac{2}{[x]} =$

(A) -2

(B) -1

(C) 0

(D) 2

(E) the limit does not exist

Calculus

1.5 Algebraic Properties of Limits and Piecewise Functions

Write your questions and thoughts here!

Notes

$$x + x =$$

$$\lim_{x \to c}[f(x) + f(x)] =$$

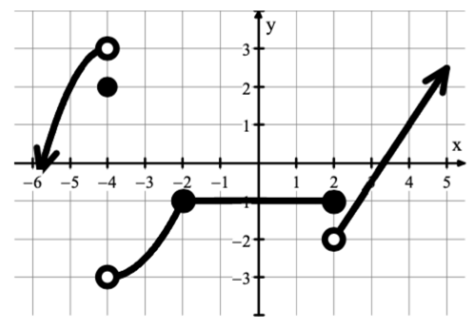
Example 1:

$\lim_{x \to -1} f(x) = 2$	$\lim_{x \to 1} f(x) = 4$	$\lim_{x \to 1} g(x) = 6$

The table above gives selected limits of the functions f and g. What is $\lim_{x \to 1}\left(f(-x) + \frac{g(x)}{2}\right)$?

Example 2:

The graph of the function f is shown on the right. What is $\lim_{x \to 4} f\big(f(x)\big)$?



Graph of f

Example 3:

$f(5) = 1$	$\lim_{x \to 5} f(x) = 6$
$g(5) = 2$	$\lim_{x \to 5} g(x) = -1$
$h(5) = 3$	$\lim_{x \to 5} h(x) = 5$

The table above gives selected values and limits of the functions f, g, and h.
What is $\lim_{x \to 5}\Big(h(x)\big(f(x) + 2g(x)\big)\Big) - h(5)$?

Example 4: Piecewise Functions

Piecewise defined functions and limits

$$f(x) = \begin{cases} \sqrt{11 - x}, & x < -5 \\ \dfrac{x+3}{5 - x^2}, & x \geq -5 \end{cases}$$

a. $\lim_{x \to -5^-} f(x) =$ b. $\lim_{x \to -5^+} f(x) =$

c. $\lim_{x \to -5} f(x) =$

$$g(x) = \begin{cases} \sqrt{10 - x^2}, & x < -1 \\ \dfrac{26 - 5x^2}{7} & -1 < x \leq e \\ \ln x^3, & x > e \end{cases}$$

a. $\lim_{x \to -1} g(x) =$ b. $\lim_{x \to e^+} g(x) =$

c. $\lim_{x \to e} g(x) =$

Calculus

1.6 Algebraic Manipulation (Limits)

Notes

Write your questions and thoughts here!

Direct Substitution		Factor and Cancel	
1. $\lim\limits_{x \to -1} (x^2 + 2x - 4)$	2. $\lim\limits_{x \to 2} 6$	3. $\lim\limits_{x \to 0} \frac{4x^2 - 5x}{x}$	4. $\lim\limits_{x \to -7} \frac{2x^2 + 13x - 7}{x + 7}$

Limit Does Not Exist

5. $\lim\limits_{x \to -6} \frac{x^2 + 4x + 3}{x + 6}$

Special Trig Limits:

$$\lim_{x \to 0} \frac{\sin x}{x} = \qquad \text{or} \qquad \lim_{x \to 0} \frac{x}{\sin x} =$$

$$\lim_{x \to 0} \frac{1 - \cos x}{x} = \qquad \text{or} \qquad \lim_{x \to 0} \frac{\cos x - 1}{x} =$$

6. $\lim\limits_{x \to 0} \frac{\sin 3x}{x}$	7. $\lim\limits_{x \to 0} \frac{\sin 7x}{\sin 9x}$	8. $\lim\limits_{x \to 0} \frac{\cos^2 x - 1}{x(\cos x + 1)}$

Calculus

1.7 Selecting Procedures (Limits)

Notes

Write your questions and thoughts here!

Rationalize Fractions with Radicals

1. $\lim_{x\to 5} \frac{\sqrt{x+4}-3}{x-5}$

2. $\lim_{x\to 10} \frac{x-10}{3-\sqrt{x-1}}$

Complex Fractions

3. $\lim_{x\to 0} \frac{x}{\frac{1}{x-4}+\frac{1}{4}}$

4. $\lim_{x\to 0} \frac{\frac{1}{(x+3)^2}-\frac{1}{9}}{x}$

Calculus

1.8 The Squeeze Theorem

Notes

Write your questions and thoughts here!

<u>Squeeze Theorem:</u> a.k.a. "Sandwich Theorem" or "Pinching Theorem"

$$\text{If } \quad (x) \leq \quad (x) \leq \quad (x)$$

$$\text{and if } \lim g(x) = \quad \text{ and } \lim h(x) =$$

$$\text{then } \lim f(x) =$$

1. Find $\lim_{x\to 0} x^2 \cos\left(\frac{1}{x^2}\right)$

2. Let g and h be the functions defined by $g(x) = -x^2 + 2x - 3$ and $h(x) = 2x + 1$. If f is a function that satisfies $g(x) \leq f(x) \leq h(x)$ for all x, what is $\lim_{x \to 2} f(x)$?

3. Let g and h be the functions defined by $g(x) = \cos\left(\frac{\pi}{2}x\right) + 2$ and $h(x) = x^2 + 3$. If f is a function that satisfies $g(x) \leq f(x) \leq h(x)$ for $-1 \leq x \leq 5$, what is $\lim_{x \to 0} f(x)$?

Let f, g, and h be the functions defined by $f(x) = \frac{1-\cos x}{2x}$, $g(x) = x^3 \sin\left(\frac{1}{x}\right)$, and $h(x) = \frac{x}{\sin x}$ for $x \neq 0$. All of the following inequalities are true for $x \neq 0$. State whether each inequality can be used with the squeeze theorem to find the limit of the function as x approaches 0?

4. $-\frac{1}{2} \leq f(x) \leq x^2 + \frac{1}{2}$

5. $-x^3 \leq g(x) \leq x^3$

6. $1 - x^2 \leq h(x) \leq x^2 + 1$

Calculus

1.9 Multiple Representations (Limits)

Notes

Write your questions and thoughts here!

1. If f is a piecewise linear function such that $\lim_{x \to 5} f(x)$ does not exist, which of the following could be representative of the function f.

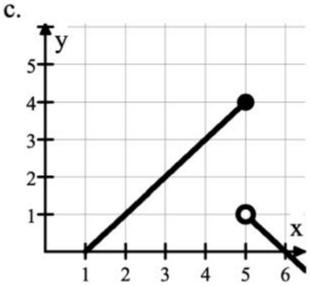
a.

$$f(x) = \begin{cases} 2x - 1, & x < 5 \\ 14 - x, & x > 5 \end{cases}$$

b.

x	2	3	4	5	6	7	8
$f(x)$	10	8	6	1	6	11	16

c.



2. Find the value of $\lim_{x \to 4^-} \frac{|x-4|}{x-4}$

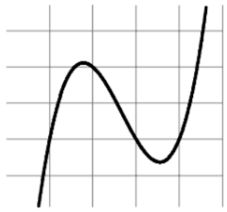
Calculus

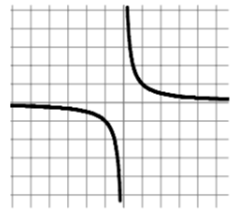
1.10 Types of Discontinuities

Notes

Write your questions and thoughts here!

<u>Continuity</u>



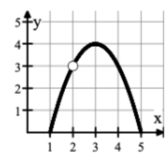


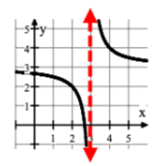
<u>Types of Discontinuities:</u>

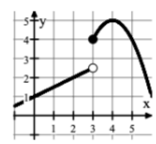
1.

2.

3.







For each function identify the type of each discontinuity and where it is located.

1. $f(x) = \frac{x^2-8x+12}{x^2+3x-10}$

2. $g(x) = \frac{x+1}{x^4-1}$

3. $h(x) = \tan 2x$ for $0 \leq x \leq 2\pi$

4. $f(x) = x^2 - 1$

1.11 Defining Continuity at a Point

Write your questions and thoughts here!

Formal Definition of Continuity:

For $f(x)$ to be continuous at $x = c$, the following three conditions must be met:

1.

2.

3.

1. State whether the function $f(x) = \begin{cases} x^2 - 2x + 1, & x < -1 \\ x + 2, & -1 \le x < 2 \\ 2^x, & x \ge 2 \end{cases}$ is continuous at the given x values. Justify your answers!

   a. $x = -1$

   b. $x = 2$

Identify the type of discontinuities (if any) and where they occur.

2. $f(x) = \begin{cases} 3x + 1, & x < 4 \\ \frac{x}{2} - 1, & x \ge 4 \end{cases}$

3. $g(x) = \begin{cases} x^2 + 2x - 1, & x < -1 \\ x - 1, & x > -1 \\ 5 & x = -1 \end{cases}$

4. Let h be the function defined by $h(x) = \begin{cases} 2 - x^2, & x \le -2 \\ 4x + k, & x > -2 \end{cases}$. What value of k would make h continuous?

Calculus

1.13 Removing Discontinuities

Notes

Write your questions and thoughts here!

Recall: What is a removable discontinuity?

$$\lim_{x \to c} f(x) \text{ exists, but } \lim_{x \to c} f(x) \neq f(c)$$

1. $f(x) = \frac{x^2-1}{x-1}$

Find the x-value of the hole.	How do we find the y-value?

2. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2+6x+8}{x+4}$ when $x \neq -4$, then $f(-4) =$

3. Let f be the function defined by $f(x) = \begin{cases} \frac{x^2-3x-18}{x-6}, & x \neq 6 \\ a, & x = 6 \end{cases}$. For what value of a is f continuous at $x = 6$?

Calculus

1.14 Infinite Limits and Vertical Asymptotes

Notes

Write your questions and thoughts here!

Use the function $f(x) = \frac{x^2+2x-8}{x^2+x-12}$ to answer the following.

1. Identify all vertical asymptotes.	2. Evaluate $\lim_{x \to 3^-} f(x)$	3. Evaluate $\lim_{x \to 3^+} f(x)$

Calculus

1.15 Limits at Infinity and Horizontal Asymptotes **Notes**

Write your questions and thoughts here!

Horizontal Asymptotes: (End-behavior)

What does the y-value approach as the x-value approaches negative infinity AND positive infinity? Does it approach a specific number, or is it growing without bound?

Basic Rules for Horizontal Asymptotes:

grows faster means $\frac{\text{not as big}}{\text{super duper BIG number!}} =$

If the numerator and denominator grow fast, then you have $\frac{\text{BIG number!}}{\text{BIG number!}} =$

If the grows faster than the denominator, then you have $\frac{\text{BIG number!}}{\text{not as big}} =$

First, you need to recognize which functions grow faster as x-values get larger and larger.

Rank Fastest to Slowest	$f(x)$	$x = 1$	$x = 10$	$x = 100$	$x = 1000$
	x^2	1	100	10,000	10^6
	x^3	1	1,000	10^6	10^9
	x^{10}	1	10^{10}	10^{20}	10^{30}
	2^x	2	1,024	1.26×10^{30}	1.07×10^{301}
	e^x	2.718	22,026	2.69×10^{43}	REALLY BIG
	4^x	4	1.05×10^6	1.6×10^{60}	SUPER-DUPER BIG
	$\ln x$	0	2.303	4.605	6.908

Find the horizontal asymptote(s) of each function.

4. $y = \frac{x^2+4}{3x-5}$

5. $y = \frac{x+4}{3x-5}$

6. $y = \frac{x+4}{3x^2-5}$

7. $f(x) = \frac{(x+5)(x-2)}{(4x-3)^2}$

8. $y = \frac{\sqrt{4x^2+x-2}}{3x-1}$

9. $y = \frac{\sqrt{4x^4+x-2}}{3x^2-1}$

Evaluate the limit.

10. $\lim_{x\to\infty} -4e^{\frac{1}{x}}$

11. $\lim_{x\to\infty} 5e^{-x}$

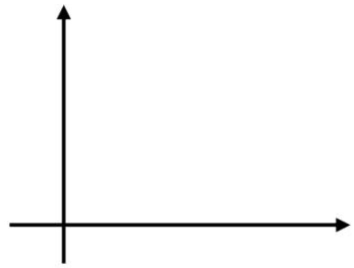
Calculus

1.16 Intermediate Value Theorem (IVT)

Notes

Write your questions and thoughts here!

Intermediate Value Theorem (for continuous functions) - **IVT**



Justification with the IVT.

1. The function $f(x)$ is continuous on an interval [   ].
2. $f(\ ) < f(\ )$ or $f(\ ) > f(\ )$.
3. $f(\ )$ is between $f(\ )$ and $f(\ )$.

Conclusion: "According to the IVT, there is a value   such that $f(\ ) =$ ___ and $\leq$ $\leq$."

Below is a table of values for a continuous function f.

x	0	3	4	8	9
$f(x)$	1	-5	3	7	-1

1. On the interval $0 \leq x \leq 9$ what is the minimum number of zeros?

2. On the interval $4 \leq x \leq 9$, what is the fewest possible times $f(x) = 1$?

3. On the interval $0 \leq x \leq 4$, ***must*** there be a value of x for which $f(x) = 2$? Explain.

4. On the interval $4 \leq x \leq 8$, ***could*** there be a value of x for which $f(x) = -2$? Explain.

5. Will the function $f(x) = x^2 - x + 1$ ever equal 8 on the interval $[-1, 5]$? Explain.

Calculus

2.1 Average and Instantaneous Rate of Change

Notes

Write your questions and thoughts here!

Recall: Rate of Change

Average rate of change on the interval $[a, b]$ is represented by

Average rate of change from a **function**.

Find the average rate of change of $f(x) = \ln 3x$ over the interval $1 \le x \le 4$.

Average rate of change from a **table**.

x	0	2	7	30
$f(x)$	3	-2	5	7

Find the average rate of change over the interval $2 \le x \le 30$.

Average Rate of Change:

The following quotients express the average rate of change of a function over an interval.

$$\frac{f(a+h)-f(a)}{(a+h)-a} \quad \text{or} \quad \frac{f(x)-f(a)}{x-a}$$

This is also the of the line.

Instantaneous Rate of Change:

The following limits express the *instantaneous* rate of change of a function at $x = a$.

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} \quad \text{or} \quad \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

This is also the of the line.

Write your questions and thoughts here!

Find the instantaneous rate of change of $f(x) = x - x^2$ at $x = -1$.

$\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

$\lim_{x \to a} \frac{f(x)-f(a)}{x-a}$

Identify the function we are working with. Then identify the x-value for the instantaneous rate of change (slope of the tangent line at a point).

1. $\lim_{h \to 0} \frac{5\ln\left(\frac{2}{4+h}\right) - 5\ln\left(\frac{1}{2}\right)}{h}$

Function: $f(x) =$

Instantaneous rate at $x =$

2. $\lim_{x \to \frac{\pi}{2}} \frac{\sin x - 1}{x - \frac{\pi}{2}}$

Function: $f(x) =$

Instantaneous rate at $x =$

Calculus

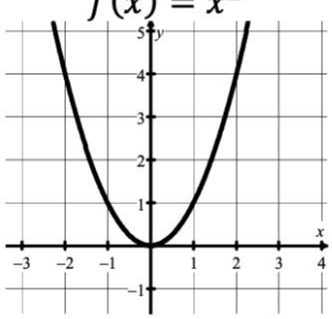
2.2 Defining the Derivative

Notes

Write your questions and thoughts here!

This ***derivative*** is an expression that calculates the instantaneous rate of change (slope of the tangent line) of a function at any given x-value. In other words, it gives us the slope of the function at a point!

$f(x) = x^2$



$f'(x) = 2x$

$f'(1) =$

$f'(2) =$

$f'(-2) =$

1. If $f'(x) = \frac{5}{x} - x$, find $f'(2)$ and explain the meaning.

2. If $f(x)$ represents how many meters you have run and x represents the minutes, describe in full sentences the following:

$f(8) = 1{,}500$	$f'(3) = 161$

Notation for the Derivative:

Lagrange	Leibniz

Defintion of the Derivative:

This limit gives an expression that calculates the instantaneous rate of change (slope of the tangent line) of $f(x)$ at any given x-value.

Write your questions and thoughts here!

Find the derivative using the Definition of the Derivative (limits).

3. $f(x) = 2x^2 - 7x + 1$

4. $y = \frac{1}{x^2}$

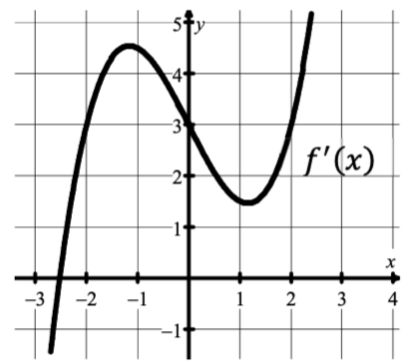
Equation of the Tangent Line:

The line tangent to the curve of $f(x)$ at $x = a$ can be represented in point-slope form:

$$y - y_1 = m(x - x_1)$$

5. If we know $h(5) = -2$, and the derivative of h is given by $h'(x) = \frac{x^3-2}{x}$, write an equation for the line tangent to the graph of h at $x = 5$.

6. The graph of $f'(x)$, the derivative of f, is shown at the right. If $f(2) = 7$, write an equation of the line tangent to the graph of f at $(2, 7)$.



Calculus

2.3 Estimating Derivatives

Notes

Write your questions and thoughts here!

Estimating the Derivative with a CALCULATOR

1. If $f(x) = \sin\sqrt{x}$, find $f'(2)$.

2. If $f(x) = \ln\left(\frac{1}{5-x}\right)$, find $f'(1.3)$.

3. Write the equation of the line tangent to $y = \sqrt{\frac{x}{x^3+1}}$ at $x = 1$.

Estimating the Derivative from TABLES

The function must be differentiable to estimate a derivative! This just means, the graph is **continuous** and **smooth**.

x hours	0	2	4	7	11
$f(x)$ miles	-2	3	10	1	-3

Using the table, estimate $f'(3)$. Show the work that leads to your answer.

x Seconds	10	50	80	120	150
$w(x)$ Gallons per second	950	850	700	500	150

Using the table, estimate $w'(100)$. Show the work that leads to your answer.

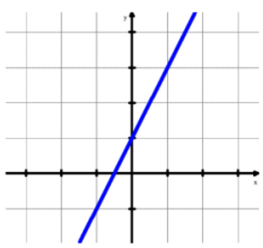
Calculus

2.4 Differentiability and Continuity

Notes

Write your questions and thoughts here!

A graph of a function is shown below. Write down its equation on line #1.



1. $y =$ ____________________
2. $y =$ ____________________
3. $y =$ ____________________

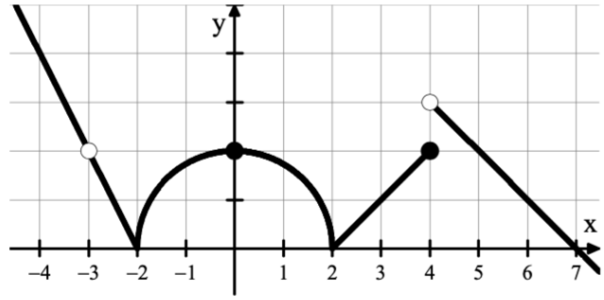
Differentiability:

The derivative exists for each point in the domain. The graph must be a smooth line or curve for the derivative to exist. In other words, the graph looks like a line if you zoom in.

The derivative ***fails to exist*** where the function has a

1.	2.	3.

Identify points where the function below is not continuous and/or not differentiable.



True or False	True or False
Differentiability implies continuity.	Continuity implies differentiability.

Calculus

2.5 The Power Rule

Notes

Write your questions and thoughts here!

Function	Function's Derivative
$f(x) = x^2$	$f'(x) =$
$f(x) = x^3$	$f'(x) =$
$f(x) = x^4$	$f'(x) =$
$f(x) = x^5$	$f'(x) =$

The Power Rule

$$f(x) = x^n$$

$$f'(x) =$$

Easy examples

1. $y = x^{37}$

2. $y = x^9$

Not as easy examples:

3.

Function	$y = \frac{1}{x}$
Rewrite	
Differentiate	
Simplify (rewrite)	

4.

Function	$y = \frac{1}{x^4}$
Rewrite	
Differentiate	
Simplify (rewrite)	

5.

Function	$y = \sqrt{x}$
Rewrite	
Differentiate	
Simplify (rewrite)	

6.

Function	$y = \sqrt[7]{x^3}$
Rewrite	
Differentiate	
Simplify (rewrite)	

Tricky examples: Simplify first, then take the derivative.

7. $f(x) = \frac{x}{\sqrt{x}}$. Find $f'(7)$

8. $f(x) = \sqrt[3]{x}(x^3)$. Find $f'(8)$

Parallel Tangent Lines

9. Let $f(x) = x^4$ and $g(x) = x^3$. At what value(s) of x do the graphs of f and g have parallel tangent lines.

Calculus

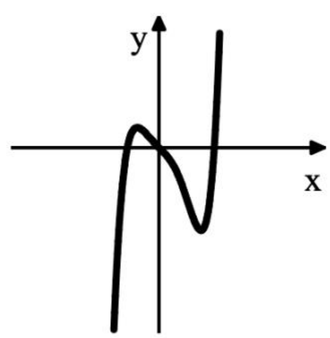
2.6 Constant, Const. Multiple, Sum/Difference

Notes

Horizontal Tangent Lines

When does a function have a horizontal tangent line?

The slope of a horizontal tangent line is zero. To find where a function has a horizontal tangent line, we set the derivative equal to zero.

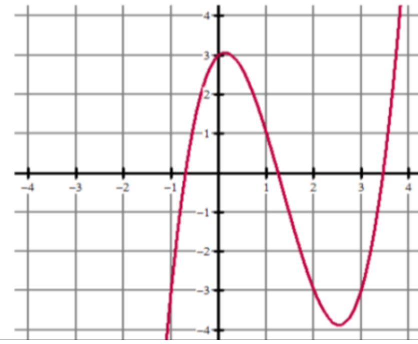


3. Find the x-values of any horizontal tangent lines of $f(x) = 4x^2 + 7x - 13$.

Normal Lines

A normal line goes through the same point the tangent line does, but it is perpendicular to the tangent line.

4. Find an equation of the NORMAL line of $f(x) = x^3 - 4x^2 + x + 3$ at $x = 3$.



Write your questions and thoughts here!

Differentiability with piecewise functions

5. Is the function $f(x) = \begin{cases} 5x^2 + 3x + 2, & x < -1 \\ -7x - 3, & x \geq -1 \end{cases}$ differentiable at $x = -1$?

6. What values of a and b would make the function $f(x) = \begin{cases} x^2 - ax + 2, & x < 3 \\ x + b, & x \geq 3 \end{cases}$ differentiable at $x = 3$?

Calculus

2.7 Derivatives of $\cos x$, $\sin x$, e^x, and $\ln x$ Notes

Write your questions and thoughts here!

Derivatives of $\cos x$ and $\sin x$

$$\frac{d}{dx}\cos x = \qquad\qquad \frac{d}{dx}\sin x =$$

Example: Find $f'(x)$ if $f(x) = 2\sin x - 5\cos x$

Recall:

$\ln 1 =$ $\qquad$ $\ln 0 =$ $\qquad$ $e^0 =$ $\qquad$ $e^{\ln a} =$ $\qquad$ $\ln e^a =$

Derivatives of Exponential Functions

$$\frac{d}{dx}a^x = \qquad\qquad \frac{d}{dx}e^x =$$

Example: Find $f'(x)$ if $f(x) = 2^x + 3e^x$

Derivatives of Logarithmic Functions

$$\frac{d}{dx}\log_a x = \qquad\qquad \frac{d}{dx}\ln x = \frac{d}{dx}\log_e x =$$

Example: Find $f'(x)$ if $f(x) = \log_4 x - 4\ln x$

Find the derivative of each function.		
1. $f(x) = 2\sin x + 5e^x$	2. $f(x) = 3^x - 4\cos x$	3. $f(x) = \log_2 x - \sin x$

Calculus

2.9 The Quotient Rule

Notes

Write your questions and thoughts here!

Quotient Rule

$$h(x) = \frac{f}{g}$$

$$h'(x) =$$

Find the derivative of each function.	
1. $y = \frac{2x^2}{3x+1}$	2. $g(x) = \frac{3e^x}{2x}$

The table below shows values of two differentiable functions f and g, as well as their derivatives.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	4	-2	-1	2

5. $h(x) = \frac{f(x)}{3g(x)}$
   Find $h'(2)$.

6. $r(x) = -\frac{g(x)}{1-f(x)}$
   Find $r'(2)$.

Calculus

2.10 Derivatives of $\tan x$, $\cot x$, $\sec x$, and $\csc x$ Notes

Write your questions and thoughts here!

Trig Derivatives

$\frac{d}{dx}\sin x =$

$\frac{d}{dx}\tan x =$

$\frac{d}{dx}\sec x =$

$\frac{d}{dx}\cos x =$

$\frac{d}{dx}\cot x =$

$\frac{d}{dx}\csc x =$

Common struggles for students dealing with trig derivatives:

- Memorizing.
- Unit Circle values.
- Simplifying/manipulating trig expressions.
- Trig reciprocals in a calculator.

1. Find the derivative of $y = \sin x \tan x$

2. Find $f'\left(\frac{\pi}{6}\right)$ if $f(x) = \frac{x}{\sec x}$

3. Estimate the derivative with a calculator of $g(x) = \csc^2 4x$ at $x = 2$

Calculus

3.1 The Chain Rule

Notes

Write your questions and thoughts here!

Composite Functions:

$$f(g(x))$$

$\sin(x^2)$	$\sqrt{\ln x}$	$\cos(\sin(5x))$

The Chain Rule (derivative of a composite function)

$$\frac{d}{dx}f(g(x)) =$$

Find the derivative

1. $f(x) = (x^2 - 5)^4$	2. $g(x) = \sqrt{4x - 3}$	3. $h(x) = \sin^2 5x$

4. $y = \ln(x^3)$	5. $y = \ln(x^3)$

6. $f(x) = \left(\frac{t^2+1}{2t-5}\right)^3$

7. If $g(x) = 2x\sqrt{1-x}$ find $g'(-3)$.

8. Given the following table of values, find $f'(4)$ for each function.

x	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
3	-1	7	-2	-3
4	3	-2	9	5

$\boldsymbol{f(x)} = (g(x))^2$

$\boldsymbol{f(x)} = \sqrt{h(x)}$

$\boldsymbol{f(x)} = h(g(x))$